REFLECTION AND TRANSMISSION PHENOMENA FOR TRANSIENT STRESS WAVES IN FIBER COMPOSITE LAMINATES

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INTRODUCTION

The problem which we investigate here arises from our earlier results [1] which are reproduced in Fig. 1. This figure shows the upper surface and lower surface normal displacement for three different fiber composite laminates due to a line load impact acting on the upper surface in each case. The laminates are constructed of layers of the same fiber composite material, perfectly bonded together. Each layer consists of a family of parallel strong elastic fibers lying in the plane of the layer and embedded in an isotropic elastic matrix. The three laminates to which the results in Figure 1 relate are a symmetric four-ply plate with $(90^0/0^0)$s configuration, a six-ply plate with layers arranged in a $(-60^0/60^0/0^0)$s configuration and a quasi-isotropic eight-ply plate with layer configuration $(90^0/-45^0/45^0/0^0)$s.

In Figure 1 each curve shows the variation of displacement with distance measured along the direction normal to the line load, in the appropriate surface, at a fixed time after the impact. For the results shown in Figure 1, the line load in each instance is taken to make an angle of -60^0 with the fiber direction in the outer layer so that the resulting plane wave propagates at an angle of 30^0 to the outer layer fibers. A comparison of the upper and lower surface displacements for each plate shows the existence of a large amplitude disturbance on the top surface of the six-ply plate and of the eight-ply plate which is not transmitted through to the bottom surface in either case. For the four-ply laminate on the other hand, the upper and lower surface displacements are similar to each other and there is no trace of any large amplitude upper surface motion such as exists for the other two plates. The position of this large amplitude motion relative to the impact location is consistent with a wave which has travelled with the speed of a surface wave on a half space of the material forming the upper layer. Earlier results [2,3,4] relating to the four-ply plate, predicted the existence of a surface wave on the upper surface for line loads aligned in one range of angles relative to the outer fibers and the non-existence of a surface wave for orientations of the line load outside this range. This phenomenon is associated with the behaviour of the dispersion curves which relate the frequency to the wavenumber for infinite trains of plane harmonic waves propagating in the laminate under traction free conditions on the outer surfaces. Because of the symmetry of the laminae, there exists two sets of these dispersion curves, one associated with motions which are symmetric with respect to the middle surface and the other relating to disturbances which are antisymmetric relative to the mid-plane.

In order to calculate the response of the plate to the impulsive surface loading, it is necessary to carry out an integration along each dispersion curve followed by a summation over all the curves (see [2] for details). The major contribution is generally

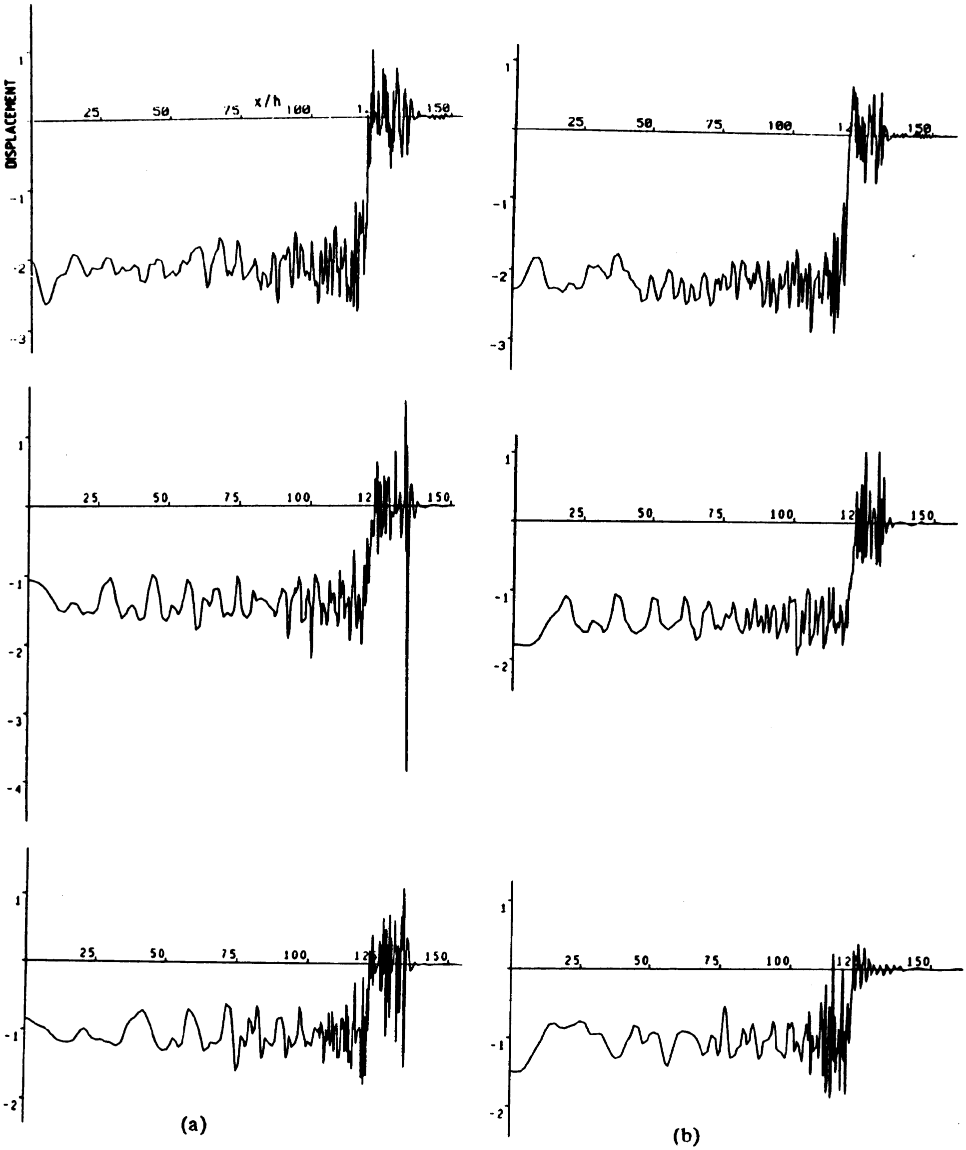

Figure 1 (a) Upper surface and (b) Lower surface displacements for wave propagation at angle 30° to the outer fiber direction for four-ply plate (top curves), six-ply plates (middle curves) and eight-ply plates (bottom curves).

associated with the fundamental mode (first branch) of each set, but results reported in [3] show that the cumulative effect of the higher modes can be significant. The curves displayed in Figure 1 have been obtained by summing over a total of eighteen modes (nine of each of the symmetric and of the antisymmetric motion) and integrating in each case over a finite range of wavenumber $0 < kh < 20$. Here, h is the individual ply thickness in each plate and k is the wavenumber along the propagation direction in the plane of the laminate. Our conclusions [2,3,4] concerning the existence and non-existence of the surface wave in the four-ply plate at various orientations of the line load are based on the limiting behaviour of the dispersion curves at short wavelengths ($kh \to \infty$). In this short wavelength limit, each of the dispersion curves asymptotes some straight line through the origin whose slope ω/k (where ω is the circular frequency) depends on the angle β between the direction of propagation and the fiber direction in the outer layers. All modes other than the fundamental mode of both symmetric and antisymmetic motion have the same asymptote. This is the line whose slope is equal to the smaller of the two body wave speeds c_{1s} and c_{2s}, where c_{1s} is the lowest quasi-shear body wave speed in the inner material and c_{2s} is the lowest quasi-shear body wave speed in the outer material at the specified angle of propagation. Since the material of both inner and outer layers is identical and the layers only differ in their fiber orientations, it is clear that the two body wave speeds c_{1s} and c_{2s} are equal for $\beta = 45^0$. For the choice of material constants employed to calculate the results shown in Figure 1 the asymptote has slope equal to c_{1s} for $\beta < 45^0$ and has slope c_{2s} for $\beta > 45^0$. In the case of the fundamental mode of both symmetric and antisymmetric motion, the asymptote is the line through the origin whose slope is the smaller of c_{1s} and c_{2r}. Here, c_{2r} is the surface wave speed in a half space of the outer material, for propagation at angle β to the fiber direction. Since $c_{2r} < c_{2s}$, there exists a critical angle $\beta_c < 45^0$ at which $c_{1s} = c_{2r}$ and for values of $\beta < \beta_c$ the fundamental modes have the same asymtotic slope, c_{1s}, as all other harmonics. For $\beta > \beta_c$, however, the two fundamental modes have asymptotic slope equal to the speed c_{2r} of surface waves on the outer material. It is this transition in asymptotic behaviour at β_c which led to our conclusion that there would be a surface wave associated with the impact for $\beta > \beta_c$, but not otherwise and our results for the four-ply plate are in accord with this expectation.

Our results concerning the asymptotic slopes of the dispersion curves may be extended to general multi-ply laminates. All curves except the fundamental mode will have asymptotic slope equal to the smallest of all the body wave speeds in the different layers, at the specified propagation angle. If the surface wave speed in the outer material at this angle is greater than this smallest body wave speed, then the fundmanetal mode will have the same asymptote as all the other modes. If the surface wave speed is less than the smallest body wave speed then the fundamental mode will asymptote the line whose slope is equal to this surface wave speed. In the latter case, we would anticipate the existence of a surface wave on the top face of the laminate in the impact problem and we would predict its non-existence in the former case. For propagation at angle $\beta = 30^0$ to the outer fibers in the four-ply plate the lowest quasi-shear wave speed in the inner material (c_{1s}) is $0.555c_1$, the lowest quasi-shear speed in the outer material (c_{2s}) is $0.678c_1$, where c_1 is a typical body wave speed in the fiber composite material. Thus, all the dispersion curves have asymptotic slope $0.555c_1$, and a surface wave would not be anticipated. For the six-ply plate, a wave propagating at an angle $\beta = 30^0$ to the outer fibers makes the same angle with the fibers of the second layer and the lowest quasi-shear wave speed in both these layers is $0.678c_1$ with the corresponding surface wave speed being $0.673c_1$. The wave propagates at angle 90^0 to the core fiber direction and the smallest quasi-shear wave speed in the core is $0.482c_1$. Thus, all dispersion curves have the same asymptotic slope of $0.482c_1$ and we do not anticipate a surface wave disturbance associated with the impact. Finally, for the eight-ply plate the lowest quasi-shear speed in the outer layer is again $0.678c_1$ with surface wave speed $0.673c_1$, the lowest quasi-shear wave speeds in the other three layers, moving inwards from the surface are $0.503c_1$, $0.718c_1$ and $555c_1$ respectively. Thus, all the dispersion curves will have asymptotic slope $0.503c_1$, associated with the third layer from the centre and a surface wave disturbance would not be predicted. The results for the six-ply and eight-ply plates, shown in Figure 1 are at variance with these predictions and are investigated in more detail in this paper.

DISCUSSION

In an attempt to understand the phenomenon described above we shall compare the behaviour of the dispersion curves associated with some of the harmonics of the four-ply plate for two different angles of propagation. We restrict attention to harmonics of the antisymmetric (flexural) modes and to propagation at angles $\beta = 30°$ and $\beta = 60°$ relative to the fiber direction in the outer layers. At the first of these angles of propagation the lowest quasi-shear body wave speed in the inner material is $c_{1s} = 0.555c_1$ and that in the outer material is $c_{2s} = 0.678c_1$, whilst the speed of a surface wave at this angle in a half space of the outer material is $c_{2r} = 0.673c_1$. For propagation at angle $\beta = 60°$, the corresponding quasi-shear body wave speeds are interchanged so that now $c_{1s} = 0.678c_1$ and $c_{2s} = 0.555c_1$ and the surface wave speed in a half space of the outer material is $c_{2r} \doteq 0.535c_1$. Thus, as remarked in the Introduction, all dispersion curves for waves travelling at $\beta = 30°$ have asymptotic slope $\omega/k = 0.555c_1$, associated with the quasi-shear body have speed in the core material. For waves travelling at $\beta = 60°$ on the other hand, the asymptotic slope of the dispersion curve for the fundamental mode is $\omega/k = 0.535c_1$, corresponding to the surface wave speed in the outer material and all other harmonics have asymptotic slope $\omega/k = 0.555c_1$, corresponding to the quasi-shear body wave speed in the outer material.

It is well-known that, for dispersive wave propagation, the parameter governing the motion of a pulse is the group velocity c_g which is given by the slope of the dispersion curve, $c_g = d\omega/dk$. In particular the wave fronts, which give rise to the dominant motion at large times, are associated with local maxima of the group velocity (advancing fronts), or with local minima of the group velocity (receding fronts). Whilst the speed of the pulse is governed by the group velocity, the nature of the disturbance is determined by the phase velocity $c = \omega/k$. To see this, consider a plane wave disturbance propagating in the xz-plane through a material in which the body wave speed is $\bar{c}$. Let the disturbance have the form $\underset{\sim}{u} = \underset{\sim}{A}\, e^{i(pz+kx-\omega t)}$, where $\underset{\sim}{A}$ is a constant vector, then the wave numbers p and k are related to the frequency ω through the expression

$$\bar{c}^2(p^2+k^2) = \omega^2 ,$$

which may be solved to express p in terms of k and the phase velocity $c = \omega/k$ in the form

$$p = k(c^2/\bar{c}^2-1)^{\frac{1}{2}} .$$

Thus for $c > \bar{c}$, p is real and the disturbance corresponds to a homogeneous plane wave propagating with speed $\bar{c}$ in the xz-plane at an angle $\alpha = \tan^{-1}(p/k)$ to the x-axis. Such a wave will undergo reflection at any interface, z = constant, which will give rise to a similar wave with p replaced by −p. The net result can be interpreted as a disturbance travelling with speed c parallel to the x-axis and having a sinusoidal variation in the z-direction. For $c < \bar{c}$, p is imaginary and the original disturbance is now an inhomogeneous wave propagating with speed c in the x-direction and having exponential decay in the z-direction. It is disturbances of this latter kind which combine together at a free surface to give rise to surface waves.

In Figure 2 we show portions of the plots of scaled phase velocity c/c_1 and scaled group velocity c_g/c_1 versus non-dimensional wave number kh for branches number 3, 4, 5 and 6 of the antisymmetric dispersion relation of the four-ply plate at $\beta = 30°$. The four upper curves relate to the phase velocity and the four lower curves show the variation of group velocity. Also shown is the line $c/c_1 = 0.678$ which corresponds to the lowest body wave speed at this angle in the outer material. As each branch of the phase velocity curves crosses from above to below this line, the nature of the associated disturbance in the outer layers changes from having a sinusoidal variation to having an exponential variation with depth but remains sinusosidal through the core. Figure 2 shows distinct plateaux formed by flattening of the phase velocity curves for branches 4, 5 and 6 almost immediately after they drop below the transition line. Associated with the plateau of each curve is a corresponding flattening of the group velocity curve for the same branch at a local maximum value. As the order of the branch increases, the plateaux of the phase velocity and group velocity curves come closer together to the extent that the curves touch for branch 6 at the speed $c/c_1 = c_g/c_1 = 0.673$, which corresponds to the speed of a surface wave in the material of the outer layers. Thus we would anticipate that the contribution to the overall disturbance from branch 6 would include a wave front travelling with the speed of the surface wave in the outer material and associated with displacements and stresses which decay with depth in the outer layer. Similar contributions but travelling with slightly lower speeds might be anticipated from each of branches 4 and 5. We note that the flat portions of both the phase velocity and the group velocity curves for one branch come to an end as the phase velocity curve of the next branch cuts the transition line. Thereafter the phase velocity curve drops smoothly towards the asymptotic value of $c/c_1 = 0.555$ whilst the group velocity curve drops almost discontinuously. We have not displayed the corresponding curves for branches 7, 8 and 9 since their transition points will occur for values of $kh > 20$ which we have taken as the cut-off point for our numerical computations. In consequence these branches will make no contribution which is associated with the speed of surface waves in the outer material. These conclusions are borne out by the results shown in Figure 3 and Figure 4. Each of these figures consists of two curves which show the variation of a typical stress component as a function of scaled distance x/h from the impact location at a fixed time $t = 40h/c_1$. The upper curve in each figure shows the stress level at the top surface of the plate whilst the lower curve gives the stress level at the upper interface. In both figures the origin of the lower curve has been displaced down to the point −5 on the vertical scale in order to display the two stresses separately. The results presented in Figure 3 show the contribution to the stress which arises from branch 6 of the antisymmetric dispersion equation whilst the plots in Figure 4 display the contribution from branch 7 of the same equation. The upper curve in Figure 3 shows a large amplitude stress contribution at $x/h \approx 26$, which corresponds to a disturbance having travelled with speed $c/c_1 \approx 0.65$. This large amplitude stress has completely decayed away on passing down through the top layer to the upper interface, which is consistent with the character of a surface wave motion. The two curves shown in Figure 4 also exhibit this phenomenon of decay of the stress level on passing down from the upper surface to the upper interface but the overall level on the top surface is considerably lower than that shown in Figure 3.

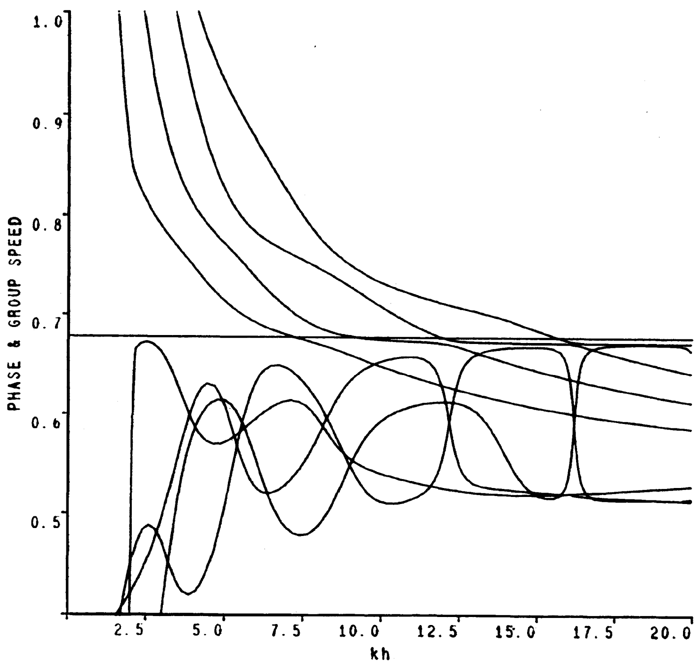

Figure 2 Phase velocity (upper curves) and group velocity (lower curves) for branches 3, 4, 5 and 6 of antisymmetric motion in four-ply plate at $\beta = 30°$.

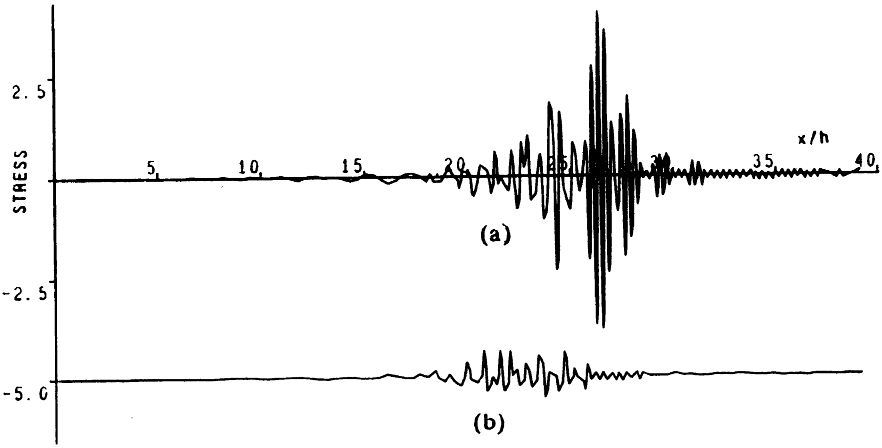

Figure 3 Stress component at time $t = 40h/c_1$, arising from branch 6 of antisymmetric motion in four-ply plate at $\beta = 30°$,
(a) Upper surface stress (b) Upper interface stress.

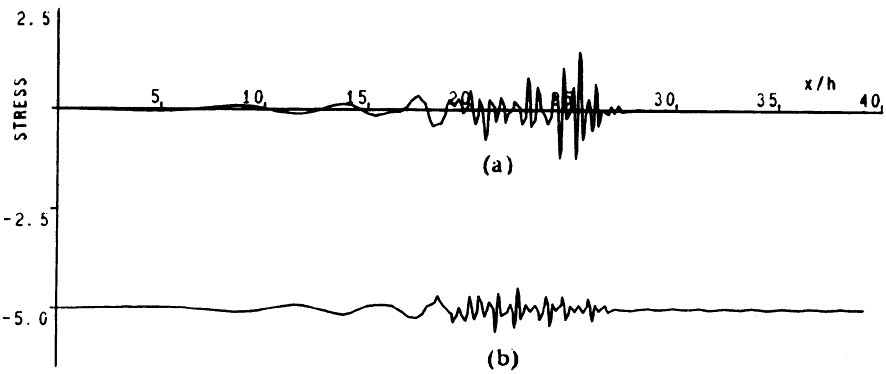

Figure 4 Stress component at time $t = 40h/c_1$, arising from branch 7 of antisymmetric motion in four-ply plate at $\beta = 30°$,
(a) Upper surface stress (b) Upper interface stress.

Figure 5 shows a set of phase and group velocity curves similar to those of Figure 2 but corresponding to waves propagating at angle $\beta = 60°$ relative to the outer fiber direction. Both sets of curves again show plateaux for branches 4, 5 and 6. In the phase velocity curves, these plateaux lie above the line $c/c_1 = 0.678$. This line is here associated with a transition of the disturbance in the inner layers from a sinusoidal to an exponential variation through the core whilst the disturbance in the outer layers remains sinusoidal throughout. The plateaux of the group velocity curves are here also coincident with local maxima and all lie below the transition line. These plateaux will therefore be associated with wave fronts but it is to be anticipated that their effect, if any, will be confined to the core material. This is borne out by the curves displayed in Figure 6, which show the stress levels at the upper surface (top curve) and at the upper interface (bottom curve) arising from branch 6 of the antisymmetric dispersion equation for $\beta = 60°$. The origin of the bottom curve has been displaced down to −5 for the sake of clarity. It is clear from these curves that, within the top layer of the plate, there is virtually no disturbance travelling with a group velocity of the order of 0.65, which is the value corresponding to the local maximum of the branch 6 curve.

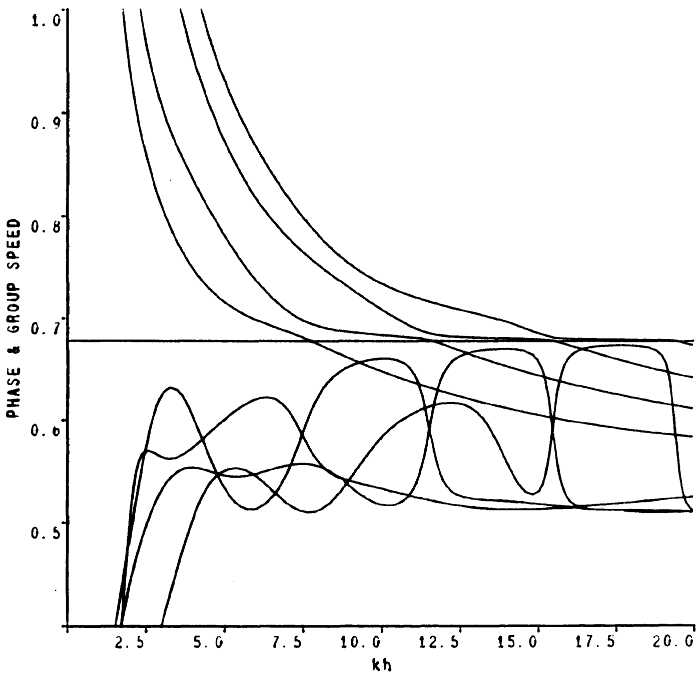

Figure 5 Phase velocity (upper curves) and group velocity (lower curves) for branches 3, 4, 5 and 6 of antisymmetric motion in four-ply plate at $\beta = 60^\circ$.

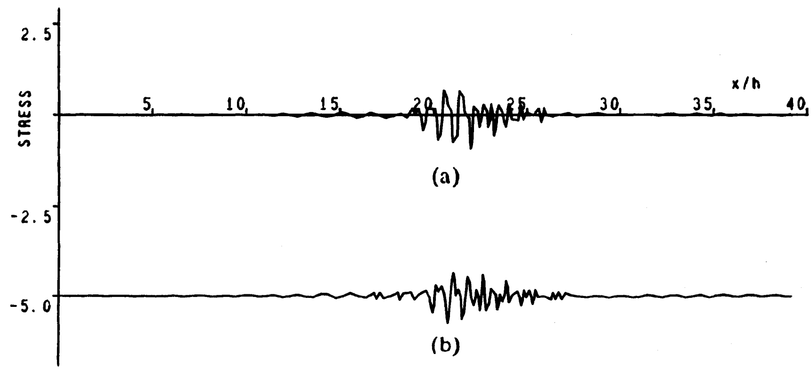

Figure 6 Stress component at time $t = 40h/c_1$, arising from branch 6 of antisymmetric motion in four-ply plate at $\beta = 60^\circ$,
(a) Upper surface stress (b) Upper interface stress.

Turning attention now to the six-ply plate, we plot the phase velocity curves and the group velocity curves for branches 3, 4, 5 and 6 of the antisymmetric dispersion equation at $\beta = 30^\circ$ in Figure 7. We remark that the plateaux of both sets of curves are now even more disinct than for the four-ply plate (Figure 2). Note also that the branch 3 curves show a relatively flat portion in this instance and that all the curves are shifted over to the left in comparison with those of Figure 2. This latter fact implies that branch 7 will also exhibit a plateau for $16.5 < kh < 20.0$. Thus branches 5, 6 and 7 will each contribute wave fronts travelling at the speed of a surface wave in the outer material, with the associated motion showing the decay with depth which is characterstic of a surface wave. It is the cumulative effect of these contributions which accounts for the surface wave displacement phenomenon exhibited in Figure 1.

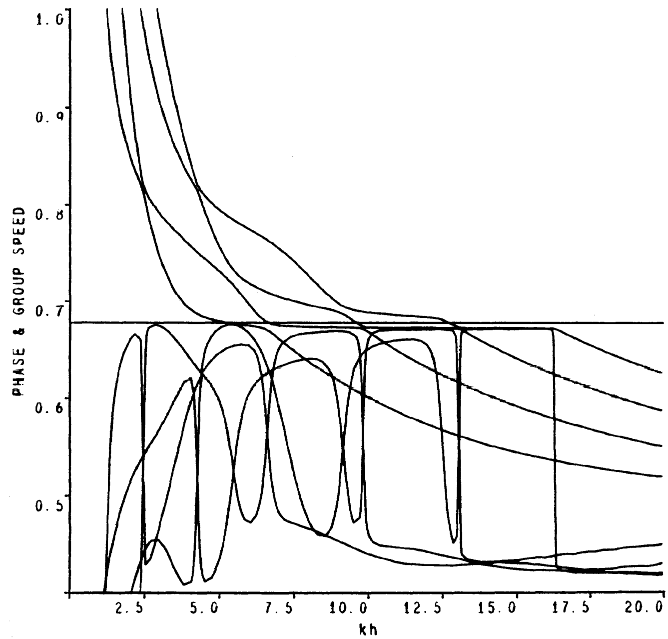

Figure 7 Phase velocity (upper curves) and group velocity (lower curves) for branches 3, 4, 5 and 6 of antisymmetric motion in six-ply plate at $\beta = 30^{\circ}$.

From these results we conclude that for waves propagating in a multi-ply plate at angle $\beta > \beta_c$ relative to the fiber direction in the outer layer, there exists a surface wave component of the disturbance which arises solely from the contribution of the fundamental mode. For waves propagating at angle $\beta < \beta_c$ on the other hand there is no surface wave effect from the fundamental mode but there exist contributions from the higher harmonics which can accumulate to give rise to a surface wave disturbance. The occurrence of the flat plateau regions which give rise to these surface wave contributions is associated with changes in the reflection/transmission properties of the interface separating the upper layer from the rest of the laminate. These changes are discussed in more detail elsewhere [5].

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